An Information-Theoretic Account of Static Index Pruning

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ABSTRACT
In this paper, we recast static index pruning as a model
induction problem under the framework of Kullback’s prin-
ciple of minimum cross-entropy. We show that static index
pruning has an approximate analytical solution in the form
of convex integer program. Further analysis on computa-
tion feasibility suggests that one of its surrogate model can
be solved efficiently. This result has led to the rediscovery
of uniform pruning, a simple yet powerful pruning method
proposed in 2001 and later easily ignored by many of us. To
empirically verify this result, we conducted experiments un-
der a new design in which prune ratio is strictly controlled.
Our result on standard ad-hoc retrieval benchmarks has con-
firmed that uniform pruning is robust to high prune ratio
and its performance is currently state of the art.

Categories and Subject Descriptors
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timeory; H.3.1 [Content Analysis and Indexing]: Indexing
methods; H.3.4 [Systems and Software]: Performance
evaluation (efficiency and effectiveness)

Keywords
Static index pruning; principle of minimum cross-entropy;
model induction; uniform pruning

1. INTRODUCTION
Kullback discussed one famous problem in his seminal
work [14] about inducing a probability measure based on
some previous measurement. When one has some initial hy-
thesis about a system and seeks to update this measure-
ment incrementally, she needs to choose a new hypothesis
from a set of feasible measures that best approximates her
current belief. Here, the difficulty lies in defining the notion
of closeness in the probability space. While at the time this
was an important issue in everyday probabilistic modeling,
a genuine solution had yet to come.

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To answer the question he had raised, Kullback introduced
a method called “minimum discrimination information,” or
in the recent literature known as the principle of minimum
cross-entropy. This approach has later become one of the
most influential inductive principles in statistics, and also
has benefited numerous fields, including some subareas in
information retrieval [15,20]. Kullback’s solution was simple
and elegant: One shall choose a measure that most closely
resembles the previous measurement in terms of Kullback-
Leibler divergence. Specifically, this is equivalent to solving
the following optimization problem, given some prior mea-
sure $p$ and a set of feasible measures $F$:

$$
\begin{align*}
\text{minimize} & \quad D(q||p) \\
\text{subject to} & \quad q \in F.
\end{align*}
$$

In this paper, we apply this induction framework to a
classic problem in information retrieval, called static index
pruning. Static index pruning is a task that reduces the in-
dex size for improving disk usage and query throughput [4].
Size reduction is done by removing index entries. Generally,
the aim in static index pruning is to find a subset of index
entries that best approximates the full index in terms of re-
trieval performance. This aspect, as we will show later, is
closely related to model induction.

One key assumption in this paper is that an inverted index
is a nonparametric, conditional distribution of document $D$
given term $T$, i.e., $p(D|T)$. This follows directly from Chen
et al.’s definition [10], which allows us to measure the resem-
blance between two versions of inverted indexes the way we
do probability distributions. Here, the following definitions
put static index pruning into the framework of Equation (1):

- The prior distribution $p$ is defined as the full (un-
  pruned) inverted index.

- The set of feasible hypotheses $F$ contains all the possi-
  ble pruned indexes of $p$ that have reached some given
  prune ratio $\rho$. In other words, each element $q \in F$ is a
  pruned version of the original inverted index $p$.

This conception marks the very beginning of our quest
for developing an efficient solution of static index pruning.
Through analysis, we first show that static index pruning
is essentially a combinatorial optimization problem. Never-
theless, in Section 3, we manage to obtain a weaker analyt-
ical solution that is practically operable by trading off some
mathematical rigor. We found that, under appropriate as-
sumptions, static index pruning reduces to a convex integer
program. But this is not a good solution in general, since the
number of variables in the convex program is linear to the
number of postings in the inverted index, which may easily exceed a few millions on any medium-sized text collection. That means this solution does not scale at all, even with the latest super-efficient convex solver.

We further attacked this problem using an alternative approach, called surrogate modeling. We created a surrogate problem that is easier to solve. As we will show in later sections, this analytical solution has pointed us to a general version of a simple pruning method called uniform pruning. Sharp-eyed readers might notice that uniform pruning is by no means a new invention. Uniform pruning was originally introduced to static index pruning in Carmel et al.’s paper as a baseline approach [9]. In a preliminary experiment, Carmel et al. compared this method with their term-based pruning method. Using TF-IDF as the score function, they found that, even though term-based method performed slightly better, in general the performance for both approaches was roughly comparable. While this was indeed a very interesting finding, the exploration was discontinued as they went ahead to study other important issues.

To the best of our knowledge, since then uniform pruning has not been studied in any follow-up work. It is easy to see why this has been the case. The lack of control on one experimental variable, prune ratio, has made the performance result difficult to interpret. When we make comparisons between methods, this variable needs to be strictly controlled so that the comparisons make sense. Nevertheless, very few in the previous work adopted this design. As a result, there was no obvious way to conduct any form of significance testing to static index pruning. Without serious scrutiny—by which we mean significance assessment—it is only reasonable to dismiss uniform pruning, for that it seemed like an ad-hoc and maybe inferior approach.

In our study, the rediscovery of uniform pruning has gained us a second chance to rethink this issue. Our answer was a redesigned empirical study, in which prune ratio for each experimental method is strictly controlled to minimize the experimental error, and the performance is analyzed using multi-way repeated-measure analysis of variance. As we will shortly cover, the experiment result suggests that uniform pruning with Dirichlet smoothing significantly outperformed the other term-based methods under diverse settings.

The rest of the paper is structured as follows. Section 2 covers an overview to static index pruning and the relevant research. In Section 3, we motivate static index pruning in the minimum cross-entropy framework and show that the analytical solution leads to the uniform pruning method. An empirical study is given in Section 4. We put the theoretical and empirical evidence together and discuss the implication in Section 5. Section 6 delivers the concluding remarks.

2. RELATED WORK

In the coming subsections, we briefly review the literature and discuss the recent development of static index pruning. Following an overview, some notable pruning methods will be treated in slightly more details. Note that this is only aimed at providing enough background knowledge for the reader. A complete coverage is not attempted here.

2.1 Overview

The idea of static index pruning first appeared in the groundbreaking work of Carmel et al. [9] and has since garnered much attention for its implication to Web-scale retrieval [11]. Static index pruning is all about reducing index size—by removing index entries from the inverted index. This technique was proposed to mitigate the efficiency issue caused by operating a large index, for that a smaller index loads faster, occupies less disk space, and has better query throughput. But since only partial term-document mapping is preserved, a loss in retrieval performance is inevitable.

Much effort has been driven towards developing importance measures of individual index entries, so that one can easily prioritize index entries on their way out of the index. Many such measures have been proposed and tested in various retrieval settings. One simple example is impact, the contribution of a term-document pair to the final retrieval score [10]. Other approaches in this line include probability ranking principle (PRP) [7], two-sample two proportion (2P2N) [9], and information preservation (IP) [10]. Some measures assess only term importance [6], so the corresponding pruning algorithms can only choose between keeping the entire term posting list or not at all. Some others assess only documents importance [21].

2.2 Methodologies

Term-based pruning (or term-centric pruning) is proposed by Carmel et al. [9]. It was so named because it attempts to reduce the posting list for each term in the index. The basic idea is to compute a cutting threshold for each term, and throw away those entries with smaller impact values. Since the cutting threshold depends on some order statistics (i.e., the k-th largest impact value) about the posting list, term-based pruning is less efficient than the other methods.

In contrast to the aforementioned term-centric approach, document-centric pruning seeks to reduce the posting list for each document. Büttcher and Clarke [8] considered the contribution for term t to the Kullback-Leibler divergence D(d||C) between document d and the collection model C. This quantity is used to measure the importance of a posting. Analogously, for each document, a cutting threshold has to be determined based on some order statistics.

There are also other pruning strategies that focus on removing an entire term posting list (whole-term) or an entire document (whole-document) all at once. Blanco and Barreiro [6] presented four term-importance measures, including inverse document frequency (idf), residual inverse document frequency (ridf), and two others based on term discriminative value (TDI). They adopted a whole-term pruning strategy. Analogously, Zheng and Cox [21] proposed an entropy-based measure in a whole-document pruning setting. Both parties have reported comparable performance to term-based pruning on some standard benchmark.

Blanco and Barreiro [7] developed a decision criterion based on the probability ranking principle [15]. The idea is to take every term in the index as a single-word query and calculate the odd-ratio of relevance p(r|t,d)/p(r|t,d). This quantity is used in prioritizing all the term-document pair. Since there is only one cutting threshold determined globally, the implementation is relatively easy and efficient.

Thota and Carterette [19] used a statistical procedure, called two-sample two-proportion (2P2N), to determine if the occurrence of term t within document d is significantly different from its occurrence within the whole collection. Chen et al. [10] developed a method called information preservation. They suggest using the innermost summand of the conditional entropy H(D|T) to measure predictive power.
contributed by individual term-document pairs to the index model. This quantity is claimed easier to compute than the probability ranking principle.

Altingovde et al. \[2\] proposed an interesting query-view technique that works orthogonal with the aforementioned measure-based approaches. The general idea is to count the number of times a document falls within the top-$k$ window of any given query collected from the query log. The count collected from a query is then evenly distributed to individual query terms. Thus the larger this number, the greater importance the posting is. The query view algorithm would later use this information to prune the entries.

Our work in this paper departs from the previous effort in three major ways. First, our approach is model-based, meaning that we infer a pruned model as a whole rather than partially. This is a novel approach in contrast to all the previous methods. Second, other information-theoretic approaches, such as Zheng and Cox \[21\] and Chen et al. \[10\], focused on minimizing the loss of information, while ours approaches, such as Zheng and Cox \[21\] and Chen et al. \[10\], the previous methods. Second, other information-theoretic approaches, such as Altingovde et al. \[2\] proposed an interesting technique that works orthogonally with the aforementioned measure-based approaches. The general idea is to count the number of times a document falls within the top-$k$ window of any given query collected from the query log. The count collected from a query is then evenly distributed to individual query terms. Thus the larger this number, the greater importance the posting is. The query view algorithm would later use this information to prune the entries.

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For now, we shall focus on estimation of this hypothetical function. The conventional approach, as discussed in Section 2.2, is to devise an importance measure to take the role of $g(\cdot)$, which is expected to capture certain properties of an index relevant to retrieval performance. Yet one caveat is that sometimes we risk being arbitrary: The importance measure may only be empirically tested and does not necessarily come with any theoretical guarantee.

One simple idea that we had failed to see casted away all these doubts. We noticed the similarity between this formulation and Kullback’s famous induction framework. As we replace $g(\cdot)$ in Equation (2) with the negative Kullback-Leibler divergence (KL divergence), the static index pruning problem reduces to a model induction problem, written in a minimization form:

$$
\begin{align*}
\text{minimize} & \quad D(\theta||\theta_0) \\
\text{subject to} & \quad \theta \subseteq \theta_0 \\
& \quad |\theta||\theta_0| \text{ reaches } 1 - \rho.
\end{align*}
$$

(3)

In the following subsections, we shall develop a procedure to practically solve this optimization problem. For brevity, we write $p(\cdot)$ and $p_0(\cdot)$, respectively, to denote the models parametrized by inverted indexes $\theta$ and $\theta_0$. The probability measures that we consider here are conditional distributions of $D$ given $T$. To make this explicit, we define:

$$
D(\theta||\theta_0) \equiv D(p(D|T)||p_0(D|T)).
$$

(4)

### 3.2 Assumptions

Before diving into the full analysis, we need to make explicit two important assumptions.

**Assumption 1 (Query and Index Models).** We can separate a joint distribution of $D$ and $T$ into a product of two models: (1) a distribution of $T$, called the query model, and (2) a conditional distribution of $D$ given $T$, called the index model. We assume there is only one query model $q(t)$ and it is independent of the index model in use. In other words, we have:

$$
p(d, t) = p(d|t)q(t), \quad p_0(d, t) = p_0(d|t)q(t).
$$

Sometimes, we simply write $p(t)$ or $p_0(t)$ to denote the query model when the meaning is clear in the context.

**Assumption 2 (Normalization Factor).** Let $p(t|d)$ and $p_0(t|d)$ be the conditional distributions of $D$ given $T$ for the induced and the original models, respectively. Let $\mathbb{I}_{t,d}$ be a binary variable that indicates whether an index entry $(t, d)$ (for some $n \in \mathbb{N}_+$) in the original model is retained in the induced model. We have $p(t|d) \equiv \mathbb{I}_{t,d}p_0(t|d)/Z_d$, where $Z_d$ is the normalization factor for document $d$.

In Assumption 2 we introduce a normalization factor $Z_d$ for each document $d$. As we shall address later, setting an appropriate value for $Z_d$ is the key step in the subsequent analysis. To correctly normalize $p(t|d)$, we need to set:

$$
Z_d = \sum \mathbb{I}_{t,d}p_0(t|d).
$$
But this would make the resulting formula intractable, since the value of \( I_{t,d} \) depends on other variables in the same document, i.e., \( I_{t,d} \). To deal with this issue, we suggest setting \( Z_d = k \) for all \( d \in D \), where \( k > 0 \) is some constant. Using this normalization trick results in weak inference and inevitably sacrifices mathematical rigors. We want to emphasize that this is a necessary compromise, without which the following analysis would not have been possible.

### 3.3 Analysis

Now, we shall go ahead and analyze the objective function. First of all, let us write out the objective in full:

\[
D(p(D|T)||p_0(D|T)) = \sum_{t,d} p(d|t) \log \frac{p(d|t)}{p_0(d|t)}
\]  

(5)

We use Assumption 1 to dissect the joint distribution \( p(d|t) \) into the product of the query model \( p(t) \) and the index model \( p_0(d|t) \). Applying Bayes Theorem to \( p(d|t) \) and \( p_0(d|t) \) and assuming uniform \( p(d) \) and \( p_0(d) \), we have the objective organized as follows:

\[
\sum_t p(t) \sum_d \frac{p_0(t|d)}{\sum_{d'} p_0(t|d')} \log \frac{p(t|d)}{p_0(t|d)} \sum_{d'} \frac{p_0(t|d')}{\sum_{d''} p_0(t|d'')}.
\]  

(6)

Observe that, since in this optimization framework we look for a subset of \( \theta \), we are essentially dealing with a combinatorial problem (“assignment problem”). Each index entry \( (t,d,n) \in \theta \) either stays within the induced model \( \theta \) or gets removed. This combinatorial nature is best characterized via the indicator variables \( I_{t,d} \) in Assumption 2.

Let us now replace all the occurrences of \( p(t|d) \). Note that, under the setting \( Z_d = k \) (suggested), all the normalization factors cancel out. We have:

\[
\sum_t p(t) \sum_d \frac{p_0(t|d)}{\sum_{d'} p_0(t|d')} \log I_{t,d} \sum_{d'} \frac{p_0(t|d')}{\sum_{d''} p_0(t|d'')}.
\]  

(7)

As we separate the support of the inner summation over \( d \) into two subsets according to whether \( I_{t,d} \) is switched on, i.e., one over \( \{d|\|d\|_2 = 1\} \) and the other over \( \{d|\|d\|_2 = 0\} \), the latter sub-summation disappears since \( 0 \log 0 = 0 \). The resulting equation becomes:

\[
\sum_t p(t) \sum_{d: I_{t,d} = 1} \frac{p_0(t|d)}{\sum_{d'} p_0(t|d')} \log \sum_{d'} \frac{p_0(t|d')}{\sum_{d''} p_0(t|d'')}.
\]  

(8)

Notice that the innermost logarithm does not depend on \( d \) anymore. We can therefore move that entire term out of the inner summation. From there, we have the inner sum over \( d \) canceled out. The equation is now written as:

\[
\sum_t p(t) \log \frac{\sum_{d'} p_0(t|d')}{\sum_{d: I_{t,d} = 1} p_0(t|d')}.
\]  

(9)

We can get rid of the numerator, i.e., \( \sum_{d'} p_0(t|d') \), in the logarithm when minimizing this equation, because the numerator does not depend any combinatorial choice we make. Once again, we rewrite it as a maximization problem by taking the negation. The final form of static index pruning is expressed as the following:

\[
\text{maximize} \quad \sum_t p_0(t) \log \sum_{I_{t,d} = 1} I_{t,d} p_0(t|d) \\
\text{subject to} \quad \sum_{t,d} I_{t,d} \text{ is binary}, \quad \text{for all } (t, d, \cdot) \in \theta, \quad \sum_{t,d} I_{t,d} = (1 - \rho)|\theta|,
\]  

(10)

Notice that this is a necessary compromise, without which the following analysis would not have been possible.

---

1In our case, the first-order Taylor expansion leads to an even more sophisticated objective.
shows that the performance of BM25 is no worse than that of a rigorously defined language model (with Jelinek-Mercer smoothing). What is left unsettled is how to estimate \( \epsilon \) given a target prune ratio \( \rho \). This issue is treated in Section 4.

4. EXPERIMENT

Thus far, we have established the theoretical ground for uniform pruning. Our next quest is to find empirical evidence that supports this result. In the coming subsections, we shall briefly describe the experiment settings and present the experimental result in greater detail.

4.1 Setup

We used three test collections in this experiment: TREC disks 4 & 5, WT2G, and WT10G. The first two collections are tested against topics 401-450 and the latter against topics 451-550. For each topic, we tested both short (title) and long (title + description) queries. Details about the benchmark are summarized in Table 1. All three collections were indexed using the Indri toolkit\(^1\). To preprocess the documents, we applied the porter stemmer and used the standard 411 InQuery stoplist. No additional text processing is done to the test collections.

According to how index traversal is preferred, a pruning method can be either term-centric or document-centric. Since different traversal strategies rely on different index creation procedures, it is difficult to have both sets implemented in one place. For simplicity, in this experiment we focused only on term-centric methods. Specifically, we tested the following methods:

1. Uniform pruning (UP)\(^2\): This method is the subject of this experiment. In this experiment, we tested three variations of uniform pruning, each using a different score function. These functions are BM25 (UP-bm25), language model using Dirichlet smoothing (UP-dir), and language model using Jelinek-Mercer smoothing (UP-jm). For BM25, we used the standard setting provided by Indri. For language models, we set \( \mu = 2500 \) in Dirichlet smoothing and \( \lambda = 0.6 \) for the Jelinek-Mercer smoother.

2. Top-\( k \) term-centric pruning (TCP)\(^3\): We set \( k = 10 \) as suggested to maximize the top-10 precision and used BM25 as the score function. Note that other score functions such as language models may also apply to this pruning method. Here, we simply comply with the previous work.

3. Probability ranking principle (PRP)\(^4\): 

\[
\frac{p(r|t,d)}{p(r|t,d)} = \frac{p(t|d)p(r|D)}{p(t|d)(1 - p(r|D))}.
\]

As suggested, we use the following equations to estimate these probabilities:

\[
p(t|D) = (1 - \lambda)p_M(t|D) + \lambda p(t|C),
\]

\[
p(r|D) = \frac{1}{2} + \frac{1}{10} \tanh \left( \frac{dl - X_d}{S_d} \right),
\]

\[
p(t|\overline{r}) = p(t|C).
\]

\(^1\)http://www.lemurproject.org/indri.php

<table>
<thead>
<tr>
<th>Collection</th>
<th># Documents</th>
<th>Query Topics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disks 4 &amp; 5</td>
<td>528k</td>
<td>401-450</td>
</tr>
<tr>
<td>WT2G</td>
<td>247k</td>
<td>401-450</td>
</tr>
<tr>
<td>WT10G</td>
<td>1692k</td>
<td>451-550</td>
</tr>
</tbody>
</table>

Table 1: Test collections and the corresponding query topics.

Note that \( dl \) is the document length. \( X_d \) and \( S_d \) are the sample mean and sample standard deviation of document length. For query likelihood, we set \( \lambda = 0.6 \).

4.2 Prune Ratio

In this experiment, we settled on 9 fixed prune levels at \( \rho = 0.1, 0.2, \ldots, 0.9 \). To control the prune ratio, comparison is only allowed between experimental runs at the same prune level. In each reference method, the true prune ratio depends on some parameter (e.g., \( \epsilon \) in TCP and PRP), which we called the threshold parameter. To reduce the index down to the right size, we employed two different approaches to determine this cutting threshold:

1. Sample percentile: Collect the prune scores on top of a sample of index entries and use the percentile estimates to determine the right cutting threshold. This is mostly useful when the prune score is globally determined. Here, we used Definition 8 from Hyndman and Fan\(^5\) to estimate percentiles.

2. Bisection: Take an interval of feasible parameter values \([a, b]\), and test-prune using the median value \((a + b)/2\). Return the current median if the test-prune reached the expected ratio; otherwise shrink the interval in half and repeat. This method is useful when the prune score for each index entry depends on the others in the same posting list, as in TCP.

Bisection requires several test-prune runs into the entire index and is therefore more time-consuming. Sample percentile needs only one pass through the index, but the resulting prune ratio can be less precise than that with the values learned using bisection. In this paper, we applied bissection to TCP to learn the parameter \( \epsilon \), and applied sample percentile to the rest of methods. Specifically, we used a sample size of 10% of the entire index. For either case, the prune ratio error is controlled to within \( \pm 0.2\% \).

\(^5\)http://www.crc.com/ftp/doclib/95/9520.pdf
4.3 Retrieval Performance

We followed Blanco and Barreiro [7] for using BM25 as the post-pruning retrieval method. Retrieval performance is measured in mean average precision (MAP) and precision-at-10 (P@10). The result on the largest set WT10G is summarized in Figure 1 (see Figures 2 and 3 at the very end of this paper for results on the smaller sets). Each figure has four sets of measure-to-query-type combinations, and the result for each combination is given both as a table on the left and a plot on the right. These combinations from top to bottom, respectively, are MAP-t, MAP-td, P@10-t, and P@10-td. Table columns and x-axes in the plots indicate prune levels, from 0.1 to 0.9 (10% to 90%). Rows and curves indicate pruning methods.

Our result shows that, at small prune levels (≤0.5), all these methods differ little in performance, and the difference at larger prune levels seems more evident. Both PRP and IP-u, whose performance was nearly identical, have consistently achieved the bottom performance in all settings. In general, the performance for the UP family and TCP is comparable, though UP-dir performed slightly better than the other. We noticed that the performance of UP-dir is also robust to high prune ratio. On WT10G, when tested under an extreme setting with 90% prune ratio, UP-dir still managed to retain 75% of the baseline MAP for short queries, and 66.7% for long queries. Of the baseline P@10, UP-dir retained 85.1% for short queries and 83.9% for long queries. Under a less aggressive setting such as 50% prune ratio, UP-dir have done even better by retaining 90.6% and 86.8% of baseline MAP, and 95.4% and 94.2% of baseline P@10, respectively for short and long queries.

4.4 Significance Tests

We further conducted an analysis of variance (ANOVA) to check if the performance difference is significant. Due to the unbalanced size of measurement, we tested each corpus independently. Here, we assume a fixed-effect, 4-way no interaction, repeated measure design, expressed as:

\[ Y_{i,j,k,l} = a_i + b_j + c_k + d_l + \epsilon_{i,j,k,l}, \]

where \( Y_{i,j,k,l} \) is the measured performance, \( a_i \) is the query-type effect, \( b_j \) the prune-level effect, \( c_k \) the method effect, and \( d_l \) the topic effect, and \( \epsilon_{i,j,k,l} \) denotes the error.

The result is covered in Table 2. Each row indicates a measure-effect combination and each column a test corpus. Test statistics, such as degrees of freedom (DF) and F-values (F), are given for every test case. We used partial eta-square (\( \eta^2_p \)) to measure the effect size [17]. We first ran an omnibus test to see if any main effect is significant. Of all three collections, all the main effects were tested significant for
Table 2: The 4-way no-interaction ANOVA result. Each cell indicates a combination of performance measure (row) and test collection (column). Degrees of freedom and F-values are given for testing all the main effects. Effect size is given in $\eta^2_p$. In our experiment, all the main effects are significant for $p < 0.001$.

<table>
<thead>
<tr>
<th>Response</th>
<th>Main Effect</th>
<th>Disks 4 &amp; 5</th>
<th>WT2G</th>
<th>WT10G</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DF</td>
<td>F</td>
<td>$\eta^2_p$</td>
<td>DF</td>
</tr>
<tr>
<td>MAP</td>
<td>Query Type</td>
<td>F(1, 5336)</td>
<td>74.10</td>
<td>.01</td>
</tr>
<tr>
<td></td>
<td>Prune Ratio</td>
<td>F(8, 5336)</td>
<td>240.30</td>
<td>.26</td>
</tr>
<tr>
<td></td>
<td>Method</td>
<td>F(5, 5336)</td>
<td>11.00</td>
<td>.01</td>
</tr>
<tr>
<td></td>
<td>Query Topic</td>
<td>F(49, 5336)</td>
<td>885.35</td>
<td>.89</td>
</tr>
<tr>
<td>P@10</td>
<td>Query Type</td>
<td>F(1, 5336)</td>
<td>66.16</td>
<td>.01</td>
</tr>
<tr>
<td></td>
<td>Prune Ratio</td>
<td>F(8, 5336)</td>
<td>105.00</td>
<td>.14</td>
</tr>
<tr>
<td></td>
<td>Method</td>
<td>F(5, 5336)</td>
<td>20.34</td>
<td>.02</td>
</tr>
<tr>
<td></td>
<td>Query Topic</td>
<td>F(49, 5336)</td>
<td>484.06</td>
<td>.82</td>
</tr>
</tbody>
</table>

Table 3: The overall result for Tukey's HSD test. For each combination of performance measure (row) and test collection (column), pruning methods are ordered in descending mean and tested for group difference. Methods that differ significantly do not share the same group label.

<table>
<thead>
<tr>
<th>Response</th>
<th>Main Effect</th>
<th>Disks 4 &amp; 5</th>
<th>WT2G</th>
<th>WT10G</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Method</td>
<td>Mean</td>
<td>Group</td>
<td>Method</td>
</tr>
<tr>
<td>MAP</td>
<td>UP-bm25</td>
<td>.204</td>
<td>a..</td>
<td>UP-dir</td>
</tr>
<tr>
<td></td>
<td>UP-dir</td>
<td>.200</td>
<td>a..</td>
<td>UP-bm25</td>
</tr>
<tr>
<td></td>
<td>TCP</td>
<td>.196</td>
<td>ab</td>
<td>TCP</td>
</tr>
<tr>
<td></td>
<td>UP-jm</td>
<td>.191</td>
<td>bc</td>
<td>UP-jm</td>
</tr>
<tr>
<td></td>
<td>PRP</td>
<td>.187</td>
<td>..c</td>
<td>IP-u</td>
</tr>
<tr>
<td></td>
<td>IP-u</td>
<td>.187</td>
<td>..c</td>
<td>PRP</td>
</tr>
<tr>
<td>P@10</td>
<td>UP-dir</td>
<td>.433</td>
<td>a..</td>
<td>UP-dir</td>
</tr>
<tr>
<td></td>
<td>TCP</td>
<td>.433</td>
<td>a..</td>
<td>TCP</td>
</tr>
<tr>
<td></td>
<td>UP-jm</td>
<td>.424</td>
<td>a..</td>
<td>UP-jm</td>
</tr>
<tr>
<td></td>
<td>UP-bm25</td>
<td>.417</td>
<td>a..</td>
<td>UP-bm25</td>
</tr>
<tr>
<td></td>
<td>PRP</td>
<td>.392</td>
<td>..b</td>
<td>IP-u</td>
</tr>
<tr>
<td></td>
<td>IP-u</td>
<td>.389</td>
<td>..b</td>
<td>PRP</td>
</tr>
</tbody>
</table>

$p < 0.001$. Further analysis shows that query type and prune method have relatively small effect sizes, suggesting that query topic and prune ratio have much greater influence on the retrieval performance than the others do.

Post-hoc tests are then called for to examine the difference caused by different factor values. Since our experimental setting involves multiple comparison, we employed Tukey's honest significance difference (HSD) to control the overall Type I error [12]. Note that since Tukey's HSD is a one-way test, only one effect is tested in each run. In the following paragraphs, we summarize the HSD results for all the main effects. Here, since our focus is on the method effect, we shall briefly cover the other three for the sake of completeness.

**Method Effect.** Table 3 summarizes this HSD result for the method effect. For each measure-corpus combination, we assigned group labels, e.g., "a" to "d", to individual methods based on the pairwise differences in their means. The difference between two methods is significant if and only if they share no common group label.

The result is briefly summarized as follows. First, the UP family and TCP consistently achieved top performance in both MAP and P@10 across different test settings. In the leading group, UP-dir delivers slightly better performance than the others. This is even more pronounced under the Web settings, in which UP-dir significantly outperformed the rest of methods in MAP (on both corpora) and in P@10 (on WT10G only). Second, the performance for the rest of UP family and TCP is in general comparable. Take UP-bm25 and TCP. The performance difference between the two is subtle: UP-bm25 was shown superior in MAP on Disks 4 & 5 but inferior in P@10 on WT2G. Third, PRP and IP-u are inferior to all the other methods. This result is consistent with our analysis on the raw performance measurements.

**Query Type Effect.** Long queries (td) achieve better performance than short queries (t), which is expected because short queries are less precise than longer ones. This difference is confirmed on all three test collections, and appears more evident in the largest set WT10G.

**Prune Ratio Effect.** Small prune levels do better than large ones in both metrics, which is also expected since more aggressive pruning results in less information in the index. According to the pairwise comparison made within the HSD test, this result is generally true except for a few small pairs such as 0.1-against-0.2. Specifically, WT10G has many such insignificant small pairs, suggesting that retrieval on larger Web collections is less sensitive to information loss.

**Topic Effect.** The result is difficult to interpret due to the size of topic pairs, e.g., topics 451-551 on WT10G has produced 4950 such pairwise comparisons. In general, only a small number of queries have significantly deviated from the average performance, meaning that most queries are designed to be about equally difficult.
5. DISCUSSION

The experiment result for uniform pruning is generally in line with our understanding to impact, much of this was contributed by the previous work in index compression and dynamic pruning. Since many ideas come from the same outlet in the indexing pruning community, it is no surprise that uniform pruning is related to many existing impact-based methods. For example, Anh et al. [3] concluded that impact-sorted indexes combined with early termination heuristics can best optimize retrieval system performance. This technique is conceptually equivalent to uniform pruning. Further work in this line investigated impact-based pruning, an application of impact-sorting to dynamic query pruning [1]. And again, this is a dynamic version of uniform pruning. Adding to these results, our analysis shows that impact-based methods are good approximate solutions to the proposed model induction problem.

One further question that invites curious eyes is why Dirichlet smoothing worked so well with uniform pruning that it significantly outperformed all the other variations on our Web benchmark WT2G and WT10G. So far the answer is still unclear to us. Here, let us discuss a few possibilities:

- BM25 might be a poor approximation to the probability \( p(t|d) \) since the framework presented in this paper was tailored specifically for language models. While this may explain why BM25 was inferior to Dirichlet smoothing in our experiments, it does not tell us why the performance for Jelinek-Mercer smoother and for BM25 were comparable.

- Another possibility is that, since parameter optimization is lacking in our experiment, we might have failed in producing the most competitive result for BM25 and Jelinek-Mercer smoother. If this theory is true, score functions will need task-specific fine-tuning in their further use. But for that to make sense, one needs to point out in what major way the role of a score function in index pruning departs from that in ordinary ad-hoc retrieval. This may point to an interesting direction for future work, but based on the evidence collected so far this claim is difficult to verify.

With the argument given in Section 3 about the convex integer program, one may argue that it is important to prevent deleting any term posting since doing so would take the objective in Equation (10) to minus infinity. In other words, an additional constraint, called “no depletion”, shall be added into the index pruning guideline. This is because, even though we do not attempt to solve the convex program, the constraint still needs to be enforced to guarantee that information loss is bounded. In this respect, it is necessary to adopt a top-k preserving strategy (i.e., skip any term posting that has less than \( k \) entries), such as the one in TCP, to avoid deleting term postings.

6. CONCLUSION

In this paper, we review the problem of static index pruning from a brand new perspective. Given the appropriate assumptions, we show that this problem can essentially be tackled within a model induction framework, using the principle of minimum cross-entropy. The theory guarantees that the induced model best approximates the full model in terms of probability divergence. We show that static index pruning can be written as a convex integer program. Yet exact inference, though possible as it might be, is generally computationally infeasible for large collections. So we further propose a surrogate model to address the computation issue, and show that uniform pruning is indeed an optimal solution to the formalism. To verify the correctness of our result, we conducted an extensive empirical study. The experiment was redesigned to take two factors, variable control and significance testing, into consideration. This setup has helped us reduce possible experimental bias or error.

Our result confirms that, when paired with the Dirichlet smoother, the performance of uniform pruning is state of the art. Significant improvement over the other methods were observed across diverse retrieval settings. Uniform pruning also exhibits an advantage in robustness with respect to large prune ratio. Specifically, our result on WT10G for short queries suggests that uniform pruning with the Dirichlet smoother retains at least 90% of the baseline performance at 50% prune ratio and 85% at 80% prune ratio. To the best of our knowledge, this is by far the best performance ever reported for static index pruning on the standard benchmark.

This research work has given rise to many technical issues, some have been addressed in Section 6 and some remain unsettled. It shall be interesting to see how uniform pruning responds to other test environments, such as different retrieval engines, corpora, or tasks. Document-length update and pseudo relevance feedback have been two landmark issues that we are ready to explore. Since we did not fine-tune the baseline performance, testing pruning methods against optimized, strong baseline shall provide more insight about this art. Besides all these possibilities, one promising direction is to extend the model induction idea to other type of structured data, such as lexicons or language models. Further investigation into the theory may shed us some light in the role that impact plays in different IR tasks.

7. ACKNOWLEDGMENT

We would like to thank Wei-Yen Day, Ting-Chu Lin, and the anonymous reviewers for their useful comments.

8. REFERENCES


MAP (t)  .1  .2  .3  .4  .5  .6  .7  .8  .9
TCP  225  221  213  206  198  186  172  149  109
UP-bm25  227  226  223  218  208  194  186  166  143
UP-dir  225  222  213  206  197  190  178  157  117
UP-jm  224  221  213  206  197  190  178  157  117
IP-u  227  225  220  211  194  168  148  149  108
IP-u  227  225  220  211  194  168  148  149  108

MAP (td) .1 .2 .3 .4 .5 .6 .7 .8 .9
TCP  251  246  239  232  219  201  184  162  117
UP-bm25  250  246  235  229  222  202  179  176  158
UP-dir  251  247  239  232  220  196  173  133
UP-jm  250  246  238  228  216  194  173  148  110
PRP  227  224  219  209  194  168  148  149  108
IP-u  227  224  219  209  194  168  148  149  108

P10 (t) .1 .2 .3 .4 .5 .6 .7 .8 .9
TCP  438  434  434  430  428  426  424  384  326
UP-bm25  438  440  436  438  438  412  370  358  294
UP-dir  438  434  432  428  434  416  398  344
UP-jm  436  442  444  452  428  400  340  292  226
PRP  436  442  444  452  428  400  340  292  226
IP-u  436  442  444  452  428  400  340  292  226

P10 (td) .1 .2 .3 .4 .5 .6 .7 .8 .9
TCP  476  468  478  474  468  460  462  432  358
UP-bm25  470  486  482  470  444  420  400  390  324
UP-dir  474  468  472  462  458  450  440  442  386
UP-jm  476  470  470  468  466  442  430  412  336
PRP  486  486  468  470  428  388  350  316  236
IP-u  474  478  474  464  436  400  340  292  226

Figure 2: The performance result on Disk 4 & 5, with all measurements rounded to the 3rd decimal place, preceding zero and decimal point removed. The best raw performance at each prune level is underlined. The unpruned baseline has has achieved 0.228/0.256 (t/td) in MAP and 0.436/0.478 (t/td) in P@10.

29th annual international ACM SIGIR conference on Research and development in information retrieval, SIGIR '06, pages 372–379, New York, NY, USA, 2006. ACM.


Figure 3: The performance result on WT2G. All measurements were rounded to the 3rd decimal place, and preceding zero and decimal point removed. The best raw performance at each prune level is underlined. The unpruned baseline has has achieved 0.249/0.293 (t/td) in MAP and 0.414/0.460 (t/td) in P@10.


